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Series (2) is equivalent to $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}$..............$\frac{n}{(n+1)!}$..............

*This becomes $\sum\limits_{n=1}^{\infty}\frac{n}{n+1!}=\sum\limits_{n=1}^{\infty}\left(\frac{1}{n!}-\frac{1}{(n+1)!}\right)=1$.

Also solved by G. B. M. Zerr, and the Proposer.

AVERAGE AND PROBABILITY.

165. Proposed by **HENRY HEATON**, Atlantic, Iowa.

What is the average length of all straight lines that can be drawn within a given square parallel to one of the diagonals?

I. Solution by **R. D. CARMICHAEL**, Hartselle, Ala.

Their intersections along the other diagonal will be evenly distributed. The average length is thus readily seen to be one-half the diagonal$=\frac{1}{2}a\sqrt{2}$, where a=the side of the square.

II. Solution by **J. EDWARD SANDERS**, Hackney, Ohio.

Since the greatest length is $a\sqrt{2}$, and the least 0, the average length is $\frac{1}{2}(a\sqrt{2}+0)=\frac{1}{2}a\sqrt{2}$; or by calculus,

$$\Delta=\frac{1}{a}\int_0^a x\sqrt{2}\,dx=\tfrac{1}{2}a\sqrt{2}.$$

Also solved by F. P. Matz, and G. B. M. Zerr.

166. Proposed by **F. P. MATZ**, Sc. D., Ph. D.

Find the average area intercepted by two non-intersecting chords drawn at random in a given circle.

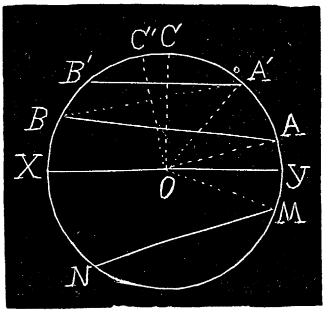
Solution by the **PROPOSER**.

First Case. The two chords AB and $A'B'$ may be on the *same* side of the diameter XY.

Let $OA=r$, $\angle OAB=\theta$, $\angle OA'B'=\phi$, $\angle YOA'=\psi$, and $\angle YOA=\omega$; then area

$$AA'B'B=U_1=(\phi-\theta+\sin\phi\cos\phi-\sin\theta\cos\theta)r^2.$$

$$\therefore\ A_1=\frac{\int_0^{\frac{1}{2}\pi}\int_\theta^{\frac{1}{2}\pi}\int_0^{2\pi}\int_\omega^{\frac{1}{2}\pi}U_1\,d\theta\,d\phi\,d\omega\,d\psi}{\int_0^{\frac{1}{2}\pi}\int_\theta^{\frac{1}{2}\pi}\int_0^{2\pi}\int_\omega^{\frac{1}{2}\pi}d\theta\,d\phi\,d\omega\,d\psi}$$



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*C. Smith, *Treatise on Algebra*, p. 396, Ex. 2.

$$=\frac{8r^2}{\pi^2}\int_0^{\frac{1}{2}\pi}(\tfrac{1}{8}\pi^2+\tfrac{1}{2}\theta^2-\tfrac{1}{2}\pi\theta+\tfrac{1}{2}\cos^2\theta-\tfrac{1}{2}\pi\sin\theta\,\cos\theta+\theta\sin\theta\,\cos\theta)d\theta=\tfrac{1}{6}\pi r^2.$$

Second Case. The two chords AB and $A'B'$ may be on *different* sides of the diameter XY; that is, the chord $A'B'$ now becomes the chord MN in position.

$\therefore$ Area $AMNB=U_2=(\phi+\theta+\sin\phi\,\cos\phi+\sin\theta\,\cos\theta)r^2$; and consequently,

$$A_2=\frac{\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}U_2 d\theta\, d\phi\, d\omega\, d\psi}{\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}d\theta\, d\phi\, d\omega\, d\psi}$$

$$=\frac{4r^2}{\pi^2}\int_0^{\frac{1}{2}\pi}(\tfrac{1}{8}\pi^2+\tfrac{1}{2}\pi\theta+\tfrac{1}{2}+\tfrac{1}{2}\pi\,\sin\theta\,\cos\theta)d\theta=\tfrac{1}{2}(\pi+1/\pi)r^2.$$

$\therefore A=\frac{1}{2}(A_1+A_2)=(\frac{1}{3}\pi+1/4\pi)r^2$, which is the required average area.

Corollary. If $r=1$, $A=\frac{113}{314}$; that is, the required average is slightly greater than one-third of the given circle.

CALCULUS.

201. Proposed by F. P. MATZ, Sc. D., Ph. D.

Solve $\int\int_0^{dy/dx}\frac{dw}{1+w^2}=0$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$\int\int_0^{dy/dx}\frac{dw}{1+w^2}=0.\quad \int\Big[\tan^{-1}w\Big]_0^{dy/dx}=0.\quad \therefore\int\tan^{-1}\frac{dy}{dx}=0.\quad \therefore\frac{dy}{dx}=0.$$

$y=C=$a constant.

202. Proposed by W. J. GREENSTREET, M. A., Editor of the Mathematical Gazette, Stroud, England.

Find the complete primitive of $y=2px+ap^2$. Regard the primitive as the equation giving the arbitrary constant, and if the primitive has equal roots discuss the equation expressing that condition.

Solution by G. W. GREENWOOD, M. A.

Differentiating with regard to x we obtain an equation which may be written

$$p\frac{dx}{dp}+2x=-2ap.$$

Integrating, we have $3p^2x+2ap^3=c$, where c is a constant. Eliminating p between this equation and the original equation, we get